Today I will do different examples i~~n each lecture~~, see solutions online and video later for more details. I hope this gives you more examples to work from.

Taylor Notes Summary:

$$T_n(x) = \sum_{k=0}^{n} \frac{1}{k!} f^{(k)}(b)(x-b)^k = f(b) + f'(b)(x-b) + \cdots + \frac{1}{n!} f^{(n)}(b)(x-b)^n$$

$$\text{ERROR} = |f(x) - T_n(x)| \le \frac{M}{(n+1)!}|x-b|^{n+1}$$

In lecture, we observed the six patterns:

$$e^z = \sum_{k=0}^{\infty} \frac{1}{k!} z^k \quad ; \quad \sin(z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} z^{2k+1} \quad ; \quad \cos(z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k)!} z^{2k} \qquad \text{, for all z.}$$

$$\frac{1}{1-z} = \sum_{k=0}^{\infty} z^k \; ; \; -\ln(1-z) = \sum_{k=0}^{\infty} \frac{1}{k+1} z^{k+1} \; ; \; \tan^{-1}(z) = \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} z^{2k+1} \quad \text{, for } -1 < \text{z} < 1.$$

Sp'13 Final Prob 8

Let $f(x) = x^2 \sin(x)$

(a) Find the Taylor series for f(x) at *b = 0*.

(b) Find the Taylor series based at *b = 0* for

$$F(x) = \int_0^x f(t)dt.$$

(c) Find the 6th Taylor Polynomial, $T_6(x)$, of $F(x)$ based at *b = 0*.

everything up to x^6 then stop

(a) $$x^2 \sin(x) = x^2 \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+1} = \boxed{\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k+3}}$$

(b) $$\int_0^x t^2 \sin(t)\,dt = \int_0^x \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} t^{2k+3}\,dt$$

$$= \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \frac{1}{(2k+4)} t^{2k+4} \Big|_0^x = \boxed{\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \frac{1}{(2k+4)} x^{2k+4}}$$

(c) $$\frac{1}{1!} \frac{1}{4} x^4 + \frac{-1}{3!} \frac{1}{6} x^6$$

$$= \boxed{\frac{1}{4}x^4 - \frac{1}{36}x^6}$$

Sp'15 Final Prob 8

Let

$$F(x) = \int_0^x \frac{t}{8+t^3}\,dt$$

(a) Find the Taylor series for F(x) at b = 0.

(b) Find the open interval of convergence.

(c) Use the series to find $F^{(11)}(0)$.

(a) $$\frac{t}{8+t^3} = t \cdot \frac{1}{8+t^3} = \frac{t}{8} \cdot \frac{1}{(1+\frac{1}{8}t^3)} = \frac{t}{8} \cdot \frac{1}{1-(-\frac{1}{8}t^3)}$$

$-1 < -\frac{1}{8}t^3 < 1$

$$= \frac{t}{8}\sum_{k=0}^{\infty}\left(-\frac{1}{8}t^3\right)^k = \frac{t}{8}\sum_{k=0}^{\infty}\frac{(-1)^k}{8^k}t^{3k}$$

$$= \sum_{k=0}^{\infty}\frac{(-1)^k}{8^{k+1}}t^{3k+1}$$

$\int$

(a) $$\boxed{F(x) = \sum_{k=0}^{\infty}\frac{(-1)^k}{8^{k+1}}\frac{1}{(3k+2)}x^{3k+2}}$$

(b) $-1 < -\frac{1}{8}t^3 < 1$

$\sqrt[3]{8 > t^3 > -8}$

$$\boxed{-2 < x < 2}$$

(c) $F(0) + F'(0)x + \dots + \frac{1}{11!}F^{11}(0)x^{11}$

when is the function @ x = 11?

@ $11 = 3k+2$

$3k = 9$

$\boxed{k=3}$

$\frac{1}{11!}F^{(11)}(0) = \frac{-1}{8^4}\frac{1}{11}$ ← plug 3 into the series

$$\boxed{F^{(11)}(0) = -\frac{1}{8^4}\left(\frac{1}{11}\right)(11!)}$$

Find $F^{(43)}(0)$

① Find $x^{43} \Rightarrow k = ?$

② Plug in k

$\underline{\quad A \quad} x^{43}$

③ $\frac{1}{43!} F^{(43)}(0)\, x^{43}$

set equal and solve

$F^{(43)}(0) = 43!\,(A)$

W'12 Final Prob 9

Let

$$g(x) = \overbrace{\frac{1}{1+x^2}}^{Q1} + \overbrace{e^{-2x^2}}^{Q2}$$

(a) Find the g(x) Taylor series based at b = 0.

(b) Find the open interval of convergence.

(c) Use the first three nonzero terms of the series from part (a) to approximate:

$$\int_0^1 \frac{1}{1+x^2} + e^{-2x^2}\,dt$$

$\boxed{Q1}$ $\quad -1 < -x^2 < 1 \rightarrow 0 \le x^2 < 1$

$$\frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{k=0}^{\infty} (-x^2)^k = \boxed{\sum_{k=0}^{\infty} (-1)^k x^{2k}}$$

$x^2 = 1$

$x = \pm 1$

$-1 < x < 1$

$\boxed{Q2}$

$$e^{-2x^2} = \sum_{k=0}^{\infty} \frac{1}{k!}(-2x^2)^k = \boxed{\sum_{k=0}^{\infty} \frac{(-1)^k 2^k}{k!} x^{2k}}$$

$-\infty < x < \infty$

(a) $g(x) = \left(\sum_{k=0}^{\infty} (-1)^k x^{2k}\right) + \sum_{k=0}^{\infty} \frac{(-1)^k 2^k}{k!} x^{2k}$

$$\boxed{g(x) = \sum_{k=0}^{\infty} \left((-1)^k + \frac{(-1)^k 2^k}{k!}\right) x^{2k}}$$

$\boxed{(b) \quad -1 < x < 1}$

has to work for both

(c) $\displaystyle\int_0^1 (1+1)x^0 + \left(-1 + \frac{(-1)(2)^1}{1!}\right)x^2 + \left(1 + \frac{2^2}{2!}\right)x^4\,dx$

$$\int_0^1 2 - 3x^2 + 3x^4\,dx = 2x - x^3 + \frac{3}{5}x^5 \Big|_0^1 = \boxed{\frac{8}{5}}$$

W'13 Final Prob 8

Let $f(x) = x^3 e^{x^2}$.

(a) Find the Taylor series for f(x) at b=0.

(b) Give the 5th Taylor polynomial.

(c) Find the Taylor series for

$$g(x) = \int_0^x t^3 e^{t^2}\,dt$$

F'16 Final Prob 9(a)(b)

Let

$$F(x) = \int_0^x e^{t^2}\,dt \quad \text{and} \quad G(x) = \int_0^x F(t)\,dt,$$

(a) Find the Taylor series for $F(x)$ at *b = 0*.

(b) Find $T_3(x)$, the 3rd Taylor polynomial, for $G(x)$ based at *b = 0*.

W'15 Final Prob 7

Let

$$A(x) = \int_0^x e^{-t^2}\,dt.$$

(a) Find the 1st and 2nd Taylor polynomials for $A(x)$ based at *b = 0.*

(b) Use the 2nd Taylor Polynomial to approximate

$$A\left(\frac{1}{2}\right) = \int_0^{1/2} e^{-t^2}\,dt.$$

(c) Use Taylor's inequality to give an upper bound on the error for the previous part (that is, on the interval $0 \leq x \leq 1/2$)