Today I will do different examples in each lecture, see solutions online and video later for more details. I hope this gives you more examples to work from.

Taylor Notes Summary:

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$$T_n(x) = \sum_{k=0}^n \frac{1}{k!} f^{(k)}(b)(x-b)^k = f(b) + f'(b)(x-b) + \dots + \frac{1}{n!} f^{(n)}(b)(x-b)^n$$

ERROR = $|f(x) - T_n(x)| \le \frac{M}{(n+1)!} |x-b|^{n+1}$

In lecture, we observed the six patterns:

$$e^{z} = \sum_{k=0}^{\infty} \frac{1}{k!} z^{k} \quad ; \quad \sin(z) = \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2k+1)!} z^{2k+1} \quad ; \quad \cos(z) = \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2k)!} z^{2k} \quad , \text{ for all } z.$$

$$\frac{1}{1-z} = \sum_{k=0}^{\infty} z^{k} \quad ; \quad -\ln(1-z) = \sum_{k=0}^{\infty} \frac{1}{k+1} z^{k+1} \quad ; \quad \tan^{-1}(z) = \sum_{k=0}^{\infty} \frac{(-1)^{k}}{2k+1} z^{2k+1} \quad , \text{ for } -1 < z < 1.$$

Sp'13 Final Prob 8
Let
$$f(x) = x^{2} \sin(x)$$

(a) Find the Taylor series based at $b = 0$.
(b) Find the Taylor series based at $b = 0$ for
 $F(x) = \int_{0}^{x} f(t) dt$.
(c) Find the 6th Taylor Polynomial) $T_{6}(x)$, of
 $F(x)$ based at $b = 0$.
(a) $x^{2} \sin(x) = x^{2} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2k+1)!} x^{2k+1} = \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2k+1)!} x^{2k+3}$
(b) $\int_{0}^{x} t^{2} \sin(t) dt = \int_{0}^{x} \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2k+1)!} t^{2k+3} dt$
 $= \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2k+1)!} \frac{1}{(2k+4)} t^{2k+4} \Big|_{0}^{x} = \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2k+1)!} \frac{1}{(2k+4)} x^{2k+4} \Big|_{0}^{x} = \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2k+4)!} \frac{1}{(2k+4)} x^{2k+4} \Big|_{0}^{x} = \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2k+4)!} \frac{1}{(2k+4)} x^{2k+4} \Big|_{0}^{x} = \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2k+4)!} \frac{1}{(2k+4)!} x^{2k+4} \Big|_{0}^{x} = \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2k+4)!} \frac{1}{(2k+4)!} x^{2k+4} \Big|_{0}^{x} = \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2k+4)!} \frac{1}{(2k+4)!} x^{4} + \sum_{k=0}^{\infty} \frac{(-1)^{k}}{(2k+4)!} \frac{1}{(2k+4)!} \frac{$

Sp'15 Final Prob 8

Let

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$$F(x) = \int_{0}^{x} \frac{t}{8+t^3} dt$$

(a) Find the Taylor series for F(x) at b = 0. (b) Find the open interval of convergence.

(c) Use the series to find
$$F^{(11)}(0)$$
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(a) $F(X) = \sum_{k=0}^{\infty} \frac{(-1)^{k}}{8^{k+1}} \frac{1}{(3^{k+2})}$

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 $(1+\frac{1}{8}t^3)$

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 $\sum_{k=0}^{\infty} \frac{(-1)^{k}}{2^{k+1}} t^{3k+1}$

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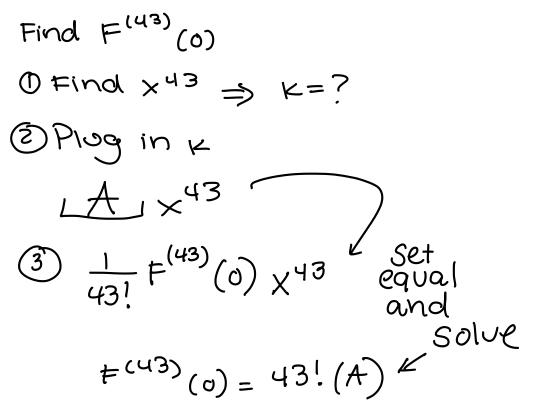
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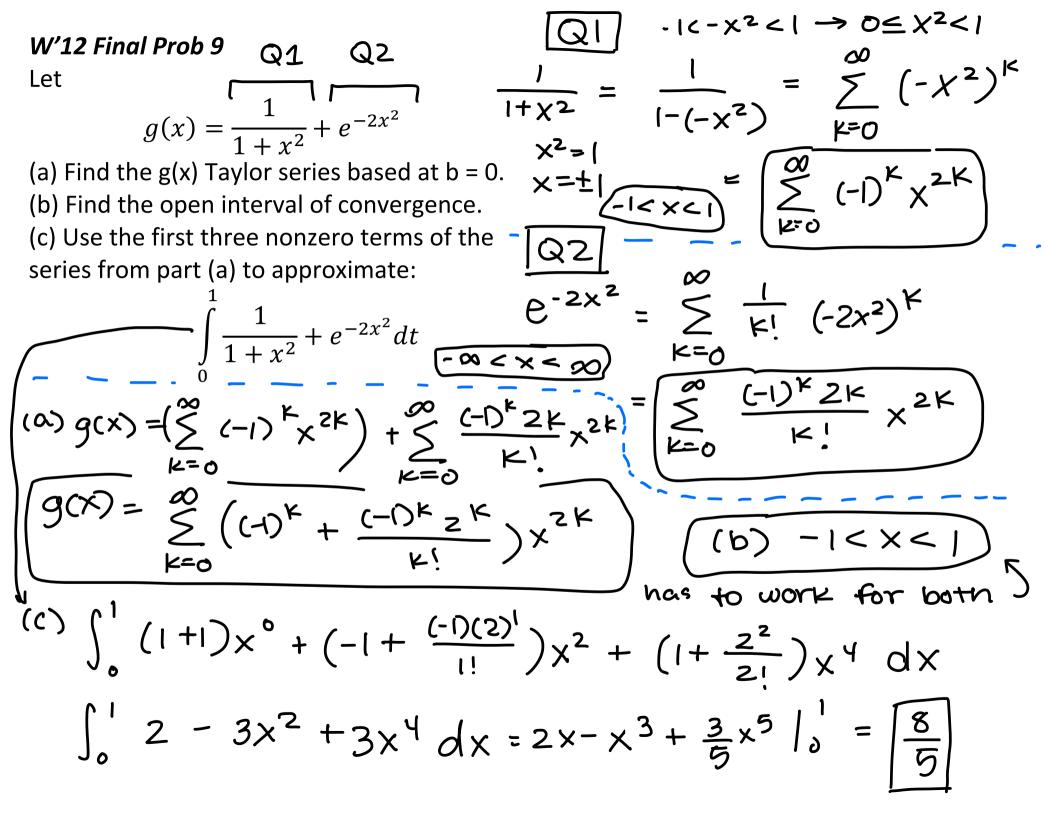
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W'13 Final Prob 8 Let $f(x) = x^3 e^{x^2}$.

(a) Find the Taylor series for f(x) at b=0.
(b) Give the 5th Taylor polynomial.
(c) Find the Taylor series for

$$g(x) = \int_0^x t^3 e^{t^2} dt$$

F'16 Final Prob 9(a)(b)

Let

$$F(x) = \int_{0}^{x} e^{t^{2}} dt$$
 and $G(x) = \int_{0}^{x} F(t) dt$,

(a) Find the Taylor series for F(x) at b = 0. (b) Find $T_3(x)$, the 3rd Taylor polynomial, for G(x) based at b = 0. W'15 Final Prob 7

Let

$$A(x) = \int_{0}^{x} e^{-t^2} dt.$$

(a) Find the 1st and 2nd Taylor polynomials for A(x) based at b = 0.
(b) Use the 2nd Taylor Polynomial to approximate

$$A\left(\frac{1}{2}\right) = \int_{0}^{1/2} e^{-t^2} dt.$$

(c) Use Taylor's inequality to give an upper bound on the error for the previous part (that is, on the interval $0 \le x \le 1/2$)